

MAS125 Calculus II 2006-2007

Problem sheet for exercise class 6

- The style of the problems is different this week, in order to give more help with the assessed coursework. There are more questions than usual but they are shorter.
- The instructor will present only one starred problem, leaving you more time to work (with help from the tutors) on the other problems (all similar to the coursework).
- Since there is not time to do all the problems in the tutorial, it is important to study the solutions once they become available online. This will be at the earlier time of 3pm Friday, giving a larger interval before the coursework deadline.

(*)Problem 1: Solve the differential equation $y' + y = x$ with $y(0) = 1$ by seeking a power-series solution of the form $y = \sum_{n=0}^{\infty} a_n x^n$ and then finding the coefficients a_n . Show that this series corresponds to the exact solution $y = 2e^{-x} - 1 + x$.

Problem 2: Find the Taylor series generated by the function $f(x)$ at $x = a$ in the following cases:

$$(a) \quad f(x) = x^3 - 2x + 4, \quad a = 2; \quad (b) \quad f(x) = 1/x^2, \quad a = 1; \quad (c) \quad f(x) = e^x, \quad a = 2.$$

Problem 3: Find the Taylor polynomials of order 1 and 2 for the following functions at $x = 0$:

$$(a) \quad f(x) = \ln(\cos x); \quad (b) \quad f(x) = 1/\sqrt{1-x^2}.$$

Problem 4: Starting with the series for $\cos x$, find the Taylor series for the following functions:

$$(a) \quad f(x) = \cos \sqrt{x+1}; \quad (b) \quad f(x) = \cos \left(\frac{x^{3/2}}{\sqrt{2}} \right); \quad (c) \quad f(x) = \sin^2 x.$$

Problem 5: Use the Remainder Estimation Theorem to estimate the error in approximating e^x by $1 + x + x^2/2$ when $|x| < 0.1$. When $-0.1 < x < 0$, show how to use the Alternating Series Estimation Theorem to estimate the error. Compare these estimations.

Problem 6: Use the Alternating Series Estimation Theorem to: (a) find for what positive values of x the function $\ln(1+x)$ can be replaced by x with a fractional error no greater than 1%; (b) demonstrate

$$1 - \frac{x^2}{6} < \frac{\sin x}{x} < 1, \quad x \neq 0.$$

Problem 7: Find the first four terms of the binomial series for the following functions:

$$(a) \quad \left(1 + \frac{x}{2}\right)^{-2}; \quad (b) \quad \left(1 + \frac{1}{x}\right)^{1/2}; \quad (c) \quad (1+x^2)^{-1/3}.$$

Problem 8: Use series to evaluate: (a) the integral below with an error of magnitude less than 10^{-8} ; (b) the limit of the function below as $x \rightarrow 0$.

$$(a) \quad \int_0^{0.1} \sqrt{1+x^4} dx, \quad (b) \quad \frac{\ln(1+x^2)}{1-\cos x}.$$